Accounting

Advance Certificate in Business Administration – Study Notes & Practice Questions

Chapter 2: Mathematics for Finance
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Introduction
Rationally, you will certainly choose the offer at the beginning of the year as the value of money makes this alternative more profitable. The concept of compounding is one of the main concepts of time value of money. The concept of compounding, in brief, explains that MVR 1 today is more valuable than MVR 1 in the future. This is because MVR 1 today can be invested to generate interest and subsequently multiply to become more than MVR 1 at the end of the investment year.

Among the reasons why the time value of money makes this alternative more valuable are:

- In general, individuals are more interested in the present usage than postponing the usage to the future.
- During the inflation periods caused by uncontrollable development in the economy, the real purchasing power of MVR 1 now is more that the real purchasing power of MVR 1 in the coming years.
- Capital that is obtained now can be invested productively to generate a higher return in the future.

The Interest Rate
Which would you prefer – MVR 1,000 today or MVR 1,000 ten years from today? Common sense tells us to take the MVR 1,000 today because we recognize that there is a time value to money. The immediate receipt of MVR 1,000 provides us with the opportunity to put our money to work and earn interest. In a world in which all cash flows are certain, the rate of interest can be used to express the time value of money. As we will soon discover, the rate of interest will allow us to adjust the value of cash flows, whenever they occur, to a particular point in time. Given this ability, we will be able to answer more difficult questions, such as: which should you prefer – MVR 1,000 today or MVR 2,000 ten years from today? To answer this question, it will be necessary to position time-adjusted cash flows at a single point in time so that a fair comparison can be made.

If we allow for uncertainty surrounding cash flows to enter into our analysis, it will be necessary to add a risk premium to the interest rate as compensation for uncertainty. In later chapters we will study how to deal with uncertainty (risk). But for now, our focus is on the time value of
money and the ways in which the rate of interest can be used to adjust the value of cash flows to a single point in time.

Most financial decisions, personal as well as business, involve time value of money considerations. In Chapter 1, we learned that the objective of management should be to maximize shareholder wealth, and that this depends, in part, on the timing of cash flows. Not surprisingly, one important application of the concepts stressed in this chapter will be to value a stream of cash flows. Indeed, much of the development of this book depends on your understanding of this chapter. You will never really understand finance until you understand the time value of money. Although the discussion that follows cannot avoid being mathematical in nature, we focus on only a handful of formulas so that you can more easily grasp the fundamentals. We start with a discussion of simple interest and use this as a springboard to develop the concept of compound interest. Also, to observe more easily the effect of compound interest, most of the examples in this chapter assume an 8 percent annual interest rate.

**Simple interest**

**Simple interest** is interest that is paid (earned) on only the original amount, or principal, borrowed (lent). The dollar amount of simple interest is a function of three variables: the original amount borrowed (lent), or principal; the interest rate per time period; and the number of time periods for which the principal is borrowed (lent). The formula for calculating simple interest is

\[ SI = P_0(i)(n) \]

where \( SI \) = simple interest in dollars  
\( P_0 \) = principal, or original amount borrowed (lent) at time period 0  
\( i \) = interest rate per time period  
\( n \) = number of time periods

For example, assume that you deposit $100 in a savings account paying 8 percent simple interest and keep it there for 10 years. At the end of 10 years, the amount of interest accumulated is determined as follows:

\[ $80 = $100(0.08)(10) \]
**Compound Interest**

The notion of **compound interest** is crucial to understanding the mathematics of finance. The term itself merely implies that interest paid (earned) on a loan (an investment) is periodically added to the principal. As a result, interest is earned on interest as well as the initial principal. It is this interest-on-interest, or *compounding*, effect that accounts for the dramatic difference between simple and compound interest. As we will see, the concept of compound interest can be used to solve a wide variety of problems in finance.

When the savings period is extended to \( t_n \), the total amount in period \( n \) is:

\[
F_n = P(1+i)^n
\]

**Calculation of Future Value using Schedule/Table**

Calculation of future value using the formula of

\[
F_n = P(1+i)^n
\]

with the value of \( n \) being more than one sometimes takes a rather long time. Therefore, the usage of a financial schedule that is the schedule of Future Value Interest Factor (FVIF \( i,n \)) helps to save time in calculations.

The future value (FV\( n \)) is equivalent to the principal at the point of time equal to 0 or the original principal amount (PV\( 0 \)) multiply with the future value factor stated in the schedule of Future Value Interest Factor (FVIF \( i,n \)). This schedule is enclosed in Attachment A.

\[
FV_n = PV_0 (FVIF_{i,n})
\]

**Example**

You deposited MVR 2,000 in the savings account in a bank at a yearly interest rate of 5% for the period of one year. Upon the completion of one year, how much will you receive?

\[
FV_n = PV_0 (FVIF_{5\%, 1})
= MVR 2,000 (1.0500)
= MVR 2,100
\]

**Example**

Assume you deposited MVR 2,000 in the savings account in your bank at a yearly interest rate of 5% for the period of four years. Upon the completion of four years, how much will you receive?
FV_n = PV_0 \times (FVIF_{i,n})
= MVR \, 2,000 \times (FVIF \, 5\%, \, 4)
= MVR \, 2,000 \times (1.216)
= MVR \, 2,432

Extract from the Future Value Interest Factor (FVIF \, i, \, n) Schedule

<table>
<thead>
<tr>
<th>Interest Rate Period no.</th>
<th>4%</th>
<th>5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.04</td>
<td>1.050</td>
</tr>
<tr>
<td>2</td>
<td>1.082</td>
<td>1.102</td>
</tr>
<tr>
<td>3</td>
<td>1.125</td>
<td>1.158</td>
</tr>
<tr>
<td>4</td>
<td>1.170</td>
<td>1.216</td>
</tr>
<tr>
<td>5</td>
<td>1.217</td>
<td>1.276</td>
</tr>
</tbody>
</table>

It must be remembered that sometimes different answer might exist by using manual calculation compared to the calculations using the schedule. This is due to the usage of different numbers of decimal points. However, the difference is not obvious and both answers are acceptable.

**Graphical Illustration of Future Value**

There are three basic elements which will influence the future value, these are:

a) Principal (amount that was borrowed or invested);

b) Time period (the number of periods or frequency of interest payments); and

c) Interest rate payable (if the money was borrowed) or interest receivable (if the money was invested).
CONCEPT OF DISCOUNTING AND PRESENT VALUE

The second concept that is related with the time value of money is the concept of cash flow discounting. This concept is used to ascertain the present value (PV0) or principal value for a sum of money in the future (FV0) that is discounted at an interest rate known as rate of return (i) for the valuation period (t). The process to determine the present value is the reverse process of determining the future value. The relationship between these two processes is illustrated in the time line.

If the discounting period is extended to tn, the principal amount that must be invested is

\[ PV_0 = \frac{FV_n}{(1+i)^n} \]

Or

\[ PV_0 = FV_n [1/(1+i)^n] \]

Calculation of Present Value (Principal) using Schedule/Table

Similar to the future value factor, the present value factor can also be obtained by using a schedule that is the Present Value Interest Factor (PVIFi,n) as attached in Attachment B. This schedule helps to simplify the calculation of present value especially in complex problems. Equation 3.4 shows that the present value (PV0) is equal to the future value amount (FVn) multiply with the present value interest factor (PVIFi,n).

\[ PV_0 = FV_n \times (PVIF i,n) \]

Example

Assume you expect to receive MVR3,999 in 3 years time from now. How much is the present value for MVR3,999 if the discount rate or rate of return is 9% per year?

\[ PV_0 = FV_n \times (PVIF 9\%,3) \]

\[ = MVR3,999 \times (0.772) \]

\[ = MVR3,087.23 \]
**Example**

You intend to accumulate MVR5,713 in a bank savings account in 4 years from now. How much savings must you deposit now if the interest rate offered by the bank is 10 percent per year?

\[ \text{PV}_0 = \text{FV}_n \times (\text{PVIF}_{i,n}) \]

\[ = \text{MVR}5,713 \times (\text{PVIF}_{10\%,4}) \]

\[ = \text{MVR}5,713 \times (0.683) \]

\[ = \text{MVR}3,901.98 \]

**Graphical Illustration of Present Value**

To show how the interest rate influences the present value (principal) of an investment, we must assume that the future value and the time period are constant. Therefore, any changes to the present value are caused only by the interest rates.

**FUTURE AND PRESENT VALUES OF A SINGLE CASH FLOW**

Single cash flow is a cash flow that only occurs once throughout the period of valuation. Both the concepts of compounding and discounting that were explained earlier have used the examples of single cash flow.
The examples stated clearly show that the future value of an amount of single cash flow invested presently will increase from time to time with the existence of specific interest rates. In reverse, a sum value of single cash flow that has been determined in the future will decrease when time approaches zero. Future value (compounding process)

FUTURE AND PRESENT VALUES OF A SERIES CASH FLOWS

The concept of future value and present value is not limited to the process of compounding and discounting single cash flow only. These concepts can be applied to a series of cash flow. A series of cash flow means that there are a series of receiving or payments of cash that occur throughout the valuation period. There are several categories of series cash flow which are annuity, derivation cash flow and perpetuity.

Annuity

Annuity is a series of payment or receiving of the same amount at the same intervals throughout the period of valuation. Annuity has a clearly stated starting point and an ending, in other words, annuity cash flow would not be indefinite. Normally, annuity occurs at the end of each period and this annuity is known as ordinary annuity. However, in some cases, annuity occurs at the beginning of the period and this type of annuity is known as annuity due.

(a) Future Value of Ordinary Annuity

Ordinary annuity is annuity that occurs at the end of each period as shown in Figure
Example

In cases where the calculation for future value of annuities are for a period of 20 or 30 years, it will be slow with complicated calculations. Therefore, we can simplify the calculations by using the formula below:

\[
FVA_n = A \left[ \frac{(1+i)^n - 1}{i} \right]
\]

\[FVA_n = A(FVIFA_{i,n})\]

Example

Aaayan Company deposited MVR5,000 at the end of each year for a period of 3 years consecutively in an account that pays a yearly interest of 10 percent. What is the future value of this annuity?

(i) Manual solution

\[
FVA_n = A \left[ \frac{(1+i)^n - 1}{i} \right]
\]

\[= MVR5,000 \left[ \frac{(1 + 0.10)^3 - 1}{0.10} \right]
\]

\[= MVR5,000 \left( \frac{1.331 - 1}{0.10} \right)
\]

\[= MVR5,000 \times 3.310
\]

\[= MVR16,550
\]

(ii) Solution using schedule

\[FVA_n = A(FVIFA_{i,n})
\]

\[= MVR5,000
\]

\[= MVR5,000 \times 3.310
\]

\[= MVR16,550
\]
(b) Future Value of Annuity Due

Sometimes we face a situation where the payment of annuity is at the beginning of a period, for example, the beginning of each month or year. This type of annuity is known as annuity due where it is different from ordinary annuity as ordinary annuity is paid at the end of a period. Annuity due occurs more frequently in future value annuity problems than present value annuity (PVA).

(i) Manual equation

\[ FVA_n = A \left( \frac{(1+i)^n - 1}{i} \right)(1+i) \]

(ii) Equation using schedule

\[ FVA_n = A \times (FVIFA_{ln})(1+i) \]

(c) Present Value of Ordinary Annuity

Payment of annuity promises a return rate (investment in bonds) and cash flow (cash flow resulting from investment in equipment and plant).

Therefore, it is important for a finance manager to know the value of the investment at the present time. The present value of ordinary annuity (PVAn) can be obtained by using the manual equation or by using the financial schedule in Attachment D. Both the equations below refer to the present value annuity (PVA n) equivalent to the annuity cash flow multiply by the present value annuity factor.

(i) Manual equation

\[ PVA_n = A \left[ \frac{1 - [1/(1+i)^n]}{i} \right] \]

(ii) Equation using schedule

\[ PVA_n = A \times (PVIFA_{ln}) \]

(d) Present Value of Annuity Due

The concept of forming equation for present value of annuity due is as per the future value of annuity due where it is based on a small alteration to the ordinary annuity equation that is by multiplying with \((1 + i)\).

\[ PVA_n = A \left[ \frac{1 - [1/(1+i)^n]}{i} \right](1+i) \]
Non-uniform Cash Flow

Lots of decision in the financial field, for example the capital budgeting and dividend payments that involve a mixture of cash flow or cash flow that is irregular. The calculation of future value and present value of an irregular cash flow is a combination concept of determining money value for single cash flow and also annuity.

(a) Future Value of Non-uniform Cash Flows

The calculation for future value of non-uniform cash flow involves the determination of future value for each of the cash flows and subsequently totalling all that future values. The future value (FVn) is obtained by adding each of the cash flow (Pt) that is adjusted with the exponent (n - t) that is the number of periods in which the interest is obtained.

Exponent is used in this formula because the last cash flow happens at the end of the last period. Therefore, interest is not obtained for it. The sigma symbol (Σ) is the mathematical symbol for a total of a series of value.

(i) Manual equation

\[ FV_n = \sum_{t=1}^{\infty} P_t (1 + i)^{n-t} \]

Example

<table>
<thead>
<tr>
<th>End of Year</th>
<th>Cash Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$5,000</td>
</tr>
<tr>
<td>1</td>
<td>$5,000</td>
</tr>
<tr>
<td>2</td>
<td>$6,000</td>
</tr>
<tr>
<td>3</td>
<td>$6,000</td>
</tr>
<tr>
<td>4</td>
<td>$1,000</td>
</tr>
</tbody>
</table>

\[
PV_0 = FV_1(PVIF_{9\%,1}) = \$5,000(0.952) = \$4,760 \\
PV_0 = FV_2(PVIF_{9\%,2}) = \$5,000(0.907) = \$4,535 \\
PV_0 = FV_3(PVIF_{9\%,3}) = \$6,000(0.864) = \$5,184 \\
PV_0 = FV_4(PVIF_{9\%,4}) = \$6,000(0.823) = \$4,938 \\
PV_0 = FV_5(PVIF_{9\%,5}) = \$1,000(0.784) = \$784 \\
\]

Present value of mixed cash flows at 5% for 5 years $20,201
(b) Present Value of Non-uniform Cash Flows

Similar to the concept in determining the future value of derivation cash flow, the present value of irregular cash flows is also a combination concept of present value of single cash flow and annuity.

Manual equation:

\[ PV_0 = \sum_{t=1}^{n} P_t \left[ \frac{1}{(1 + i)^t} \right] \]

COMPOUNDING AND DISCOUNTING MORE THAN ONCE A YEAR

The practice of compounding or discounting interest more than once a year is also known as intrayear compounding or discounting. For example, compounding or discounting twice a year, three times a year, four times a year or each month. The frequency of compounding or discounting several times in a year is a normal practice in making financial decisions.

When the frequency of compounding or discounting for future value or present value is more than once a year, the time period will become \((n \times m)\), the interest rate must also be divided with the said frequency \((i / m)\). The purpose is to adjust the changes in the period and interest rate to enable both the variables to change consistently. Therefore, a little alteration must be made to the formulas that had been learned previously.

The formula for manual solution is:

\[ FV = PV \times (1 + i/m)^{nm} \]

Where the solution by schedule is as follow:

\[ FV = PV \times (FVIF_{i/m,nm}) \]

Example

The future value of RM1 now after 6 years, using the interest rate of 10% per year with different compounding frequencies.

<table>
<thead>
<tr>
<th>Presumed Compounding</th>
<th>( nm )</th>
<th>( i / m )</th>
<th>( FV_{nm} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Once a year</td>
<td>6 x 1 = 6</td>
<td>0.1/1 = 0.1</td>
<td>RM1.772</td>
</tr>
<tr>
<td>Twice a year</td>
<td>6 x 2 = 12</td>
<td>0.1/2 = 0.05</td>
<td>RM1.796</td>
</tr>
<tr>
<td>Four times a year</td>
<td>6 x 4 = 24</td>
<td>0.1/4 = 0.025</td>
<td>RM1.809</td>
</tr>
<tr>
<td>Every month</td>
<td>6 x 12 = 72</td>
<td>0.1/12 = 0.0083</td>
<td>RM1.817</td>
</tr>
</tbody>
</table>
CONTINUOUS COMPOUNDING AND DISCOUNTING

Before this, you were only exposed to situations where the interest is compounded or discounted at specific discrete intervals whether yearly or twice a year, monthly and so forth. However, in some cases of time value of money, interest must be compounded or discounted continuously or at each micro-second. The new formula for future value and present value that is compounded and discounted continuously is as follows.

Future value: \[ FV_n = PV_0 \left( e^{in} \right) \]

Present value: \[ PV_0 = FV_n \left( e^{-in} \right) \]

The estimate number for the symbol e in equation is 2.72 (or more accurately, 2.71828183).
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PRACTICE QUESTIONS

Question 1
Roza deposits MVR100 in the savings account at Bank of Maldives with an interest rate of 5% per year for 5 years. How much would Roza have in the savings account at the end of the 5 year period?

Question 2
You want MVR1,100 in your account a year from now. How much investment must you make now if the interest rate offered by the bank is 10%?

Question 3
Ahmedbey company private limited, offers a low risk security that promises a payment of MVR3,000 at the end of 2 years period with an offer of 15% interest rate per year. What is the present value for MVR3,000?

Question 4
Use the schedule of present value interest factor to help you solve the questions below:
1. Assume that you are given the opportunity to purchase a low risk security that promised a payment of MVR127.63 at the end of 5 years with an interest rate of 5% per year. How much is the present value for MVR 127.63?
2. You plan to accumulate MVR6,213 in a bank savings account 5 years from now. How much savings must you deposit now if the interest rate offered by the bank is 12% per year?

Question 5
Solve the questions below by using the manual formula or schedule (FVIFA i,n).
1. Assume that you deposit MVR100 into the bank at the beginning of the year for 3 years in the savings account that gives 5% interest rate. How much can be obtained at the end of the third year?
2. Mr. Yashau deposits MVR10,000 into the bank on 31st December each year for 5 years at an interest rate of 10%. How much can he obtain at the end of the fifth year?
Question 8
You are offered an annuity payment of MVR100 at the end of each year for 3 years and is deposited into the bank. The interest rate offered is 5% per year. How much is the present value of that annuity payment?

Question 9
Consider the perpetuity that pays MVR100 per year, with an interest rate of 10%. How much is the present value of this perpetuity?

Question 10
What is the future value for MVR260 that is invested now for 3 years at the interest rate of 10 percent per year and compounded continuously?

Question 11
What is the present value for MVR200 that will be received 5 years from now and discounted continuously at the interest rate of 6 percent per year?

Question 13
Shehenaz Company is considering an investment on a new machine that involves a total purchase and assembly cost of MVR30,000. The usage of this new machine is expected to generate a yearly cash flow for 5 consecutive years: end of first year MVR4,000, end of second and third year MVR5,000, end of fourth year MVR6,000 and end of fifth year MVR8,000. If the company requires a yearly 18% rate of return on its investment, is it reasonable for the company to continue with its investment?

Question 14
Compute the present value for a series of indefinite yearly payments of MVR180, assuming that the interest rate is:
   a) 5 percent
   b) 10 percent
**Question 15**
You have just won a puzzle contest where you were offered two choice of prizes that is whether to accept MVR60,000 today or MVR12,000 at the end of each year for 5 consecutive years. If the cash flow is discounted at a yearly rate of 12 percent and compounded twice a year, which choice would you choose?

\[
PVOA = \text{RM12,000} \times (PVIFA_{6\%,10}) \\
= \text{RM12,000} \times (7.3601) \\
= \text{RM88,321.20}
\]

The second choice should be chosen (RM88,321.20) as the present value is more compared to the first choice (RM60,000).

**Question 16**
Mrs. Rifshana plans to get a loan for a total of MVR6,000 at the interest rate of 10% from a kind-hearted money lender. The money lender agrees to receive a sum of payment for the same amount at the end of each year for 4 years. What is the size of payment that Mrs. Rifshana must give to the money lender each year?

**Question 17**
What is the present value for MVR400 that will be received in 7 years from now and discounted continuously at the interest rate of 10 percent per year?

**Question 18**
A firm borrows $1,000 at 8 percent simple interest with all payments due at the end of four years. What amount must be repaid?

**Question 19**
If you have a choice to earn simple interest on $10,000 for three years at 8% or compound interest at 7.5% for three years which one will pay more and by how much?

**Question 20**
You deposit $1,000 into a bank account. If the bank pays 4 percent simple interest, how much will you accumulate in your account after 10 years?
Question 21

You deposit $1,000 into a bank account. If the bank pays 4 percent compound interest, how much will you accumulate in your account after 10 years?

Question 22

A credit union recently advertised an investment certificate that pays 1.75% in the first year, 2.25% in the second year, 2.75% in the third year, 3.25% in the fourth year and 5.25% in the fifth and final year. If you invest $10,000 in this investment certificate and hold it to maturity, how much will you have at the end of five years?

Question 23

Suppose someone invests $1,000 today for a five-year term and receives 10 percent annual compound interest. How much would the investor have after five years?

Question 24

You have deposited $1,500 in an account that promises to pay 8% compounded quarterly for the next five years. How much will you have in the account at the end?

Question 25

An investor estimates that she needs $1 million to live comfortably when she retires in 40 years. How much does she have to invest today, assuming a 10 percent interest rate on the investment?

Question 34

If you wish to accumulate MVR140,000 in 13 years, how much must you deposit today in an account that pays an annual interest rate of 14%?
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**Question 35**
What will MVR247,000 grow to be in 9 years if it is invested today in an account with an annual interest rate of 11%?

**Question 36**
How many years will it take for MVR136,000 to grow to be MVR468,000 if it is invested in an account with an annual interest rate of 8%?

**Question 37**
If you wish to accumulate MVR197,000 in 5 years, how much must you deposit today in an account that pays a quoted annual interest rate of 13% with semi-annual compounding of interest?

**Question 38**
What will MVR153,000 grow to be in 13 years if it is invested today in an account with a quoted annual interest rate of 10% with monthly compounding of interest?

**Question 39**
How many years will it take for MVR197,000 to grow to be MVR554,000 if it is invested in an account with a quoted annual interest rate of 8% with monthly compounding of interest?

**Question 40**
You are offered an annuity that will pay MVR24,000 per year for 11 years (the first payment will occur one year from today). If you feel that the appropriate discount rate is 13%, what is the annuity worth to you today?

**Question 41**
If you deposit MVR16,000 per year for 12 years (each deposit is made at the end of each year) in an account that pays an annual interest rate of 14%, what will your account be worth at the end of 12 years?
Question 42
You are offered an annuity that will pay MVR17,000 per year for 7 years (the first payment will be made today). If you feel that the appropriate discount rate is 11%, what is the annuity worth to you today?

Question 43
If you deposit MVR15,000 per year for 9 years (each deposit is made at the beginning of each year) in an account that pays an annual interest rate of 8%, what will your account be worth at the end of 9 years?

Question 44
You plan to accumulate MVR450,000 over a period of 12 years by making equal annual deposits in an account that pays an annual interest rate of 9% (assume all payments will occur at the beginning of each year). What amount must you deposit each year to reach your goal?

Question 45
You plan to buy a car that has a total "drive-out" cost of MVR25,700. You will make a down payment of MVR3,598. The remainder of the car’s cost will be financed over a period of 5 years. You will repay the loan by making equal monthly payments. Your quoted annual interest rate is 8% with monthly compounding of interest. (The first payment will be due one month after the purchase date.) What will your monthly payment be?

Question 46
What will a deposit of MVR4,500 at 10% compounded semiannually be worth if left in the bank for six years?

Question 47
What will a deposit of MVR4,500 at 7% annual interest be worth if left in the bank for nine years?
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Question 48
What will a deposit of MVR4,500 at 12% compounded monthly be worth at the end of 10 years?

Question 49
How much will MVR1,000 deposited in a savings account earning an annual interest rate of 6 percent be worth at the end of 5 years?

Question 50
How much will MVR1,000 deposited in a savings account earning a compound annual interest rate of 6 percent be worth at the end of 3 years?